

Exercise 34Calculate y' .

$$y = 10^{\tan \pi \theta}$$

SolutionIntroduce a logarithm in order to bring θ out of the exponent.

$$\begin{aligned} y' &= \frac{d}{d\theta} \left(10^{\tan \pi \theta} \right) \\ &= \frac{d}{d\theta} \left(e^{\ln 10^{\tan \pi \theta}} \right) \\ &= \frac{d}{d\theta} \left[e^{(\tan \pi \theta) \ln 10} \right] \\ &= e^{(\tan \pi \theta) \ln 10} \cdot \frac{d}{d\theta} [(\tan \pi \theta) \ln 10] \\ &= e^{\ln 10^{\tan \pi \theta}} \cdot (\ln 10) \left[\frac{d}{d\theta} (\tan \pi \theta) \right] \\ &= 10^{\tan \pi \theta} \cdot (\ln 10) \left[(\sec^2 \pi \theta) \cdot \frac{d}{d\theta} (\pi \theta) \right] \\ &= 10^{\tan \pi \theta} \cdot (\ln 10) [(\sec^2 \pi \theta) \cdot (\pi)] \\ &= (\pi \ln 10) 10^{\tan \pi \theta} \sec^2 \pi \theta \end{aligned}$$